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COMPARATIVE STUDY OF NEW PROPOSED METHOD FOR SOLVING ASSIGNMENT PROBLEM WITH THE EXISTING METHODS

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ABSTRACT

Assignment Problem is a special type of problem in mathematics and has anextensivescope of applications in real physical world. In this paper, I attempt to introduce a new alternate solution method for assignment problems with mathematical formulation and procedure, followed by a numerical problem by using this method and two existing methods. This new method is a systematic procedure and easy to apply in any normal case of assignment problem.

KEYWORDS

An assignment problem, Hungarian assignment method(HA-method), Matrix one's assignment method (MOA-method),new projected method, Optimization.

INTRODUCTION

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operational research in mathematics. It is a special case of the transportation problem, which is a special case of the minimum cost flow problem, which in turn is a special case of a linear program. In a normal case of assignment problem, the main objective is to find the optimum allocation of a number of jobs (resources) to an equal number of facilities (persons). In this paper, I developed a new method for solving assignment problems. The corresponding method has been formulated mathematically and numerical example has been considered to explain the method and finally, I compare the optimal solutions of the problem among new method and two existing methods.

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MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM

The assignment problem can be stated in the form of $n \times n$ matrix $[c_{ij}]_{n\times n}$ which is called costmatrix or effectiveness matrix, where c_{ij} is the cost of assigning ith facility to the jth job. The cost matrix $[c_{ij}]$ is given as

Jobs ⟶

Person ↓

$$\begin{pmatrix} & 1 & 2 & \dots & j & \dots & n \\ 1 & C_{11} & C_{12} & \dots & C_{1j} & \dots & C_{1n} \\ 2 & C_{21} & C_{22} & \dots & C_{2j} & \dots & C_{2n} \\ & ----- & & & & & \\ i & C_{i1} & C_{i2} & \dots & C_{ij} & \dots & C_{in} \\ n & C_{n1} & C_{n2} & \dots & C_{nj} & \dots & C_{nn} \end{pmatrix}$$

An assignment problem can matical form as follows

Minimize total cost

$$z = \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

where $x_{ij} = \begin{cases} 1, & \text{if ith person is assigned jth job} \\ 0, & \text{if the ith person is not assigned jth job} \end{cases}$

subject to the conditions:

(i) $\sum_{i=1}^{n} x_{ij} = 1$,j=1,2,… ,n which means that one job is done by the ith person.

(ii) $\sum_{j=1}^{n} x_{ij} = 1$,i=1,2,… ,nwhich means that only one person should be assigned to the jth job.

NEW APPROACH FOR SOLVING ASSIGNMENT PROBLEM

A new approach is introduced for solving assignment problem with the help of HA-method and MOA-method but different from them. This method is easy to apply for solving assignment problem and has the following procedure:

STEP-1: Find the least cost of each column. Subtract this smallest number from every number in that column.

STEP-2: Now add 1 to all elements and we get at least one ones in each column. Then make assignment in terms of ones. If any row and column have no assignment, then we cannot get the optimum solution. So, we proceed to the next step.

STEP-3: Draw the least possible number of lines passing through all ones by using the following procedure:

(i) Label$(\surd)$ rows that do not have assignments.

(ii) Label $(\surd)$ columns that have crossed ones in that marked rows.

(iii) Label $(\surd)$ rows that have assignments in marked columns.

(iv) Repeat (ii) and (iii) till no more rows or columns can be marked.

(v) Mark straight lines through all unmarked rows and marked columns.

If the number of lines drawn is equal to the order of matrix, then the current solution is optimal solution. Otherwise go to next step.

STEP-4: Select the least cost of the reduced matrix not covered by the lines. Divide all uncovered numbers by this least cost. Other numbers covered by lines remain unchanged and we get some new ones in row and column. Again make assignment in terms of ones.

STEP-5: If any row or column do not have any optimal assignment, then repeat steps (3) and (4) successively till an optimum solution is obtained.

NUMERICAL COMPARISON OF EXISTING METHODS WITH PROPOSED METHOD:

1. Examine the following assignment problem using new projected method

Consider the problem of assigning 5 jobs to 5 persons. The assignment costs are given below:

Jobs →

Person ↓

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

Find the optimum assignment schedule and minimum assignment cost.

SOLUTION: I. Find the minimum element of each column and subtract it from each element in that column. Then the reduced column matrix is as follows:

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	I	II	III	IV	V
A	0	2	0	2	4
B	1	3	1	0	3
C	4	5	1	3	4
D	2	0	2	1	0
E	0	4	4	4	2

II. Now add 1 to all elements.

	I	II	III	IV	V
A	1	3	1	3	5
B	2	4	2	1	4
C	5	6	2	4	5
D	3	1	3	2	1
E	1	5	5	5	3

III. Now make initial assignment by using ones.

	I	II	III	IV	V
A	[1]	3	×	3	5
B	2	4	2	[1]	4
C	5	6	[2]	4	5
D	3	[1]	3	2	×
E	×	5	5	5	3

Here, 5th row and 5th column do not have any assignment. Hence the solution is not optimum and we go to next.

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IV. Select the smallest number of the reduced matrix not covered by the lines. Divide all uncovered elements by this smallest number. Other numbers covered by lines remain unchanged. Then make assignment again.

	I	II	III	IV	V
A	[1]	~~1~~	~~1~~	~~1~~	1.6
B	2	4	2	[1]	4
C	5	2	[2]	1.3	1.6
D	3	[1]	3	2	~~1~~
E	~~1~~	1.6	5	1.6	[1]

Here all the ones are either assigned or crossed out. That is, number of assignments is equal to number of rows or columns. Therefore, the solution is optimal and the solution is (A , I) , (B , IV), (C , III), (D , II), (E , V). So, the total minimum cost is 1+1+3+1+4=10.

2. Examine the following assignment problem using Hungarian Assignment method.

Consider the problem of assigning 5 jobs to 5 persons. The assignment costs are given below:

Jobs →

Persons ↓

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

Find the optimum assignment schedule and minimum assignment cost.

SOLUTION: I. Subtract the smallest element of each row from every element of thecorresponding row, we get the following row reduced matrix.

	I	II	III	IV	V
A	0	2	1	2	5
B	1	3	2	0	4
C	2	3	0	1	3
D	2	0	3	1	1
E	0	4	5	4	3

II. Subtract the smallest element of each column from every element of the corresponding column, we get the following column reduced matrix.

	I	II	III	IV	V
A	0	2	1	2	4
B	1	3	2	0	3
C	2	3	0	1	2
D	2	0	3	1	0
E	0	4	5	4	2

III. Make the initial assignment with the help of zeroes.

	I	II	III	IV	V
A	[0]	2	1	2	4
B	1	3	2	[0]	3
C	2	3	[0]	1	2
D	2	[0]	3	1	~~0~~
E	~~0~~	4	5	4	2

Since the number of assignments is not equal to number of rows or columns. So, the solution is not optimal and we proceed to the next step.

IV. In this step, we draw minimal number of lines to cover all the zeroes at least once.

	I	II	III	IV	V
A	[0]	2	1	2	4
B	1	3	2	[0]	3
C	2	3	[0]	1	2
D	2	[0]	3	1	~~0~~
E	~~0~~	4	5	4	2

V. Select the smallest element that do not have a line through it. Subtract this element from all the elements that do not have a line through them and add it to every element that lies in the intersection of two lines.

	I	II	III	IV	V
A	~~0~~	1	~~0~~	1	3
B	2	3	2	[0]	3
C	2	3	[0]	1	2
D	2	[0]	3	1	~~0~~
E	[0]	3	4	3	1

Again the number of assignments is not equal to the number of rows or columns. So, we repeat the step IV and V.

	I	II	III	IV	V
A	~~0~~	1	~~0~~	1	3
B	2	3	2	[0]	3
C	2	3	[0]	1	2
D	2	[0]	3	1	~~0~~
E	[0]	3	4	3	1

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	I	II	III	IV	V
A	[0]	~~0~~	~~0~~	~~0~~	2
B	3	3	3	[0]	3
C	2	2	[0]	~~0~~	1
D	3	[0]	4	1	~~0~~
E	~~0~~	2	4	2	[0]

Since number of assignments is equal to the number of rows or columns. Therefore, the solution is optimal and the solution is (A, I), (B , IV), (C , III), (D , II), (E , V). So, the total minimum cost is 1+1+3+1+4=10.

3. Examine the following assignment problem using Matrix One's Assignment method.

Consider the problem of assigning 5 jobs to 5 persons. The assignment costs are given below:

Jobs →

Persons ↓

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	5	6	3	4	6
D	3	1	4	2	2
E	1	5	6	5	4

Find the optimum assignment schedule and minimum assignment cost.

SOLUTION: I. Find the minimum element of each row and then divide every element of the corresponding row by the minimum element.

	I	II	III	IV	V
A	1	3	2	3	6
B	2	4	3	1	5
C	1.6	2	1	1.3	2
D	3	1	4	2	2
E	1	5	6	5	4

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II. Find the minimum element of each column and then divide every element of the corresponding column by the minimum element.

	I	II	III	IV	V
A	1	3	2	3	3
B	2	4	3	1	2.5
C	1.6	2	1	1.3	1
D	3	1	4	2	1
E	1	5	6	5	2

III. Make assignments with the help of ones.

	I	II	III	IV	V
A	1	3	2	3	3
B	2	4	3	1	2.5
C	1.6	2	1	1.3	~~1~~
D	3	1	4	2	~~1~~
E	~~1~~	5	6	5	2

Since the number of assignments is not equal to number of rows or columns. So, the solution is not optimal and we proceed to the next step.

IV. Draw the minimal number of lines to cover all the ones at least once.

	I	II	III	IV	V
A	1	3	2	3	3
B	2	4	3	1	2.5
C	1.6	2	1	1.3	~~1~~
D	3	1	4	2	~~1~~
E	~~1~~	5	6	5	2

V. Select minimum element from uncovered elements and divide the other uncovered elements by this minimum element and then mark assignments.

	I	II	III	IV	V
A	[1]	1.5	~~1~~	1.5	1.5
B	2	4	3	[1]	2.5
C	1.6	2	[1]	1.3	~~1~~
D	3	[1]	4	2	~~1~~
E	~~1~~	2.5	3	2.5	[1]

Since number of assignments is equal to the number of rows or columns. Therefore, the solution is optimal and the solution is (A, I) , (B , IV), (C , III), (D , II), (E , V). So, the total minimum cost is 1+1+3+1+4=10.

COMPARISON OF OPTIMAL VALUES OF THREE METHODS

Example	HA-method	MOA-method	Proposed method	Optimum solution
01	10	10	10	10

Therefore, I conclude that this new proposed method is effective for solving assignment problem.

CONCLUSION

In this paper, I presented a new solution method for assignment problems. Firstly, I explained the procedure of proposed method and then showed its efficiency by numerical problem. And I get the optimum solution which is same as in case of HA-method and MOA-method. So, this paper introduces a different approach for solving assignment problem which is easy to understand and apply.

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